4. S. I. Anisimov and Ya. B. Zel'dovich, "Rayleigh–Taylor instability of the boundary between the detonation products and the gas in a spherical explosion," Pis'ma Zh. Tekh. Fiz., 3, Issue 20 (1977).
5. F. A. Goldsworthy, "The structure of a contact region with application to the reflection of a shock from a heat-conducting wall," J. Fluid Mech., No. 7 (1969).
6. G. Z. Gershuni and E. M. Zhukhovitskii, Convective Instability of an Incompressible Fluid [in Russian], Nauka, Moscow (1972).
7. S. Z. Belen'kii and E. S. Fradkin, "Theory of turbulent mixing," Trans. Fiz. Inst. Akad. Nauk (Lebedev), 29 (1965).
8. V. E. Neuvazhaev and V. G. Yakovlev, "Toward a theory of turbulent mixing of the boundary between fluids in the field of gravity," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1976).
9. V. E. Neuvazhev, "Toward a theory of turbulent mixing," Dokl. Akad. Nauk SSSR, 222, No. 5 (1975).

NONLINEAR WAVES AND STABILIZATION OF TWO-DIMENSIONAL INSTABILITY IN A BOUNDARY LAYER

V. P. Reutov

UDC 532.526.530.182

As a rule, the transition to turbulence in a boundary layer is associated with the growth of two-dimensional waves [1-4]. Consequently, the investigation of the nonlinear stage in the development of a two-dimensional instability has an important role in the creation of a transition theory. Methods of the theory of a weak nonlinearity permit computation of the coefficients in the dynamic equations proposed by Landau [5, 6] for the weak wave amplitude. However, for those values of the flow parameters that are ordinarily realized in the transition region, the weak nonlinearity approximation describes only the initial stage of wave amplification. Substantially nonlinear structures that originate in the boundary layer because of the constraint of the two-dimensional instability are examined in this paper.

The mechanism of boundary-layer instability in the case of infinitesimally small perturbations has long been studied (see [1, 7], say). It is known that the occurrence of a viscous near-wall layer (VNWL) results in wave destabilization, while resonance wave-flow interaction can attenuate or totally suppress this instability. In the case when the thickness of the resonance domain of flow interaction with the wave, the critical layer (CL) is sufficiently small, there is a possibility of analytical investigation of the substantially nonlinear stage in development of the instability [8-12]. Simplification of the problem is associated with localization of the nonlinearity within the limits of a thin CL. However, formation of the VNWL was not taken into account in [8] (slip conditions were posed at the wall). The shift in the primary flow velocity near the wall was not taken into account in [9] in the determination of the VNWL structure, which is only justified for very large Reynolds numbers. Moreover, it follows from the solution of the nonstationary problem [12] that the natural waves constructed in [9] correspond to the threshold of strict origination of instability (and are not constrained by it). Below, we solve the problem of stationary waves originating for moderately large Reynolds numbers that are characteristic for the main part of the boundary layer neutral curve loop. The analysis is constructed within the framework of CL theory and is based on graphic representations of the CL structure and the instability mechanism. From the formal point of view, the procedure proposed for the solution can be considered a generalization of the Tollmien method used to construct the neutral curve in the linear theory of hydrodynamic instability [1]. The results of computations are compared with known experimental data.

Gor'kii. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 35-42, July-August, 1985. Original article submitted April 24, 1984.

1. BYPASSING THE RESONANCE POINT

We use the vortex transport equations in a viscous incompressible fluid [1] as the starting point. We introduce normalized variables by taking the length and velocity scales equal to some characteristic boundary-layer thickness δ and the flow velocity u_∞ at infinity, respectively. The x axis of the rectangular coordinate system is directed downstream, and the y axis along the normal to the surface. Then the two-dimensional flow equations take the form

$$\frac{\partial\zeta}{\partial t}+u\frac{\partial\zeta}{\partial x}+v\frac{\partial\zeta}{\partial y}=\frac{1}{\mathrm{Re}}\Delta\zeta; \tag{1.1a}$$

$$\Delta\Psi=-\zeta\left(u=\frac{\partial\Psi}{\partial y},\ v=-\frac{\partial\Psi}{\partial x}\right), \tag{1.1b}$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$; ζ is the flow vorticity; Ψ is the stream function; u and v are the longitudinal and transverse velocity components, respectively; $\mathrm{Re} = u_\infty\delta/\nu$ is the Reynolds number, which is assumed large (ν is the kinematic viscosity of the medium). The velocity profile of the parallel flow in the boundary layer $\bar{u}(y)$ is shown in Fig. 1. We seek the solution of (1.1) in the form of stationary waves when Ψ and ζ depend only on two variables: y and $\xi = x - ct$ (c is the wave phase velocity). Representing the stream function in the form $\Psi = \int\bar{u}(y)dy + \psi$, we examine the linear equation for ψ, which follows from (1.1) in the limit case of an inviscid medium. The Rayleigh equation [4, 7], which contains a singularity at the resonance point $y = y_c$, where $\bar{u}(y_c) = c$ (see Fig. 1), as is known, is obtained here for the profile of the harmonic wave perturbations. We take the Tollmien functions [8-10], which have the form $\varphi_a = \eta + O(\eta^2)$, $\varphi_b = 1 + O(\eta^2) + (\bar{u}''_c/\bar{u}'_c)\,\varphi_a \ln|\eta|$ ($\bar{u}'_c = d\bar{u}/dy_c$, $\bar{u}''_c = d^2\bar{u}/dy^2_c$) as $\eta = y - y_c \to 0$, as basis functions for the Rayleigh equation. We consequently obtain for ψ an expression of the form

$$\psi=[A_\pm\varphi_a(y)+B_\pm\varphi_b(y)]e^{i\alpha\xi}+\text{c.c.},$$

where α is the wave number; $A_\pm$ and $B_\pm$ are constants, the + and − signs refer to the domains $y > y_c$ and $y < y_c$, respectively; and c.c. denotes complex conjugate. It is seen from (1.2) that as $y \to y_c$, the vorticity and nonlinearity fluctuations grow without limit.* Therefore, it is impossible to neglect the viscosity and nonlinearity simultaneously in the neighborhood of the resonance point. In a more general formulation (taking account of the nonstationarity of the perturbations), the problem examined here of bypassing the resonance point is analogous to that known for waves in a collisionless plasma. It is known that even in the case of an ideal fluid, the presence of resonance results in absorption of hydrodynamic waves, which can be considered as the analog of Landau damping in a plasma [13]. The problem of bypassing the singularity in linear theory reduces to the problem of selecting the branch of the multivalued function during passage through the resonance point. Analysis of the CL structure is necessary for derivation of the bypassing rule in the case of nonlinear waves. It is shown in [9] that the bypassing rule for a viscous nonlinear CL can be expressed, exactly as in linear theory, interms of a jump in the phase of the logarithm in the definition of φ_b. However, this phase jump now depends on the wave amplitude. In writing the bypassing rule for a viscous nonlinear CL in terms of the coefficients in (1.2), we have

$$B_+=B_-\equiv B,\quad A_+-A_-=-i\Phi\frac{\bar{u}''_c}{\bar{u}'_c}B, \tag{1.3}$$

where Φ is the increase in phase of the logarithm during passage from $y_c + 0$ to $y_c - 0$. If the singularity is eliminated by viscosity, the characteristic CL thickness equals $d_\ell = (\alpha|\bar{u}'_c|\mathrm{Re})^{-1/3}$ [1]; if there is no viscosity, but the nonlinearity is retained, we obtain the scale of the nonlinear CL: $d_n = (B_m/|\bar{u}''_c|)^{1/2}$ [9], where $B_m = 2|B|$ is the amplitude of the oscillations of the stream function in the CL. For a viscous and nonlinear CL we introduce the generalized scale $d_c = \max(d_c, d_n)$. Let us emphasize that these deductions are valid for flows with thin CL when $\bar{u}$, $\bar{u}'$, and $\bar{u}''$ vary slightly in the CL scale.

A graphic method of calculating the jump in phase of the logarithm Φ is given by the vortical treatment of hydrodynamic motion. Since the CL is a thin vortical layer, we replace (1.1b) by the one-dimensional

*The nonlinearity parameters as $\mathrm{Re} \to \infty$ are the ratios $(u - \bar{u})/(\bar{u} - c)$ and $v/(\bar{u} - c)$.

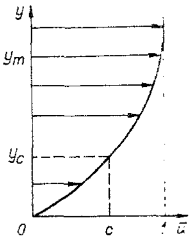

Fig. 1

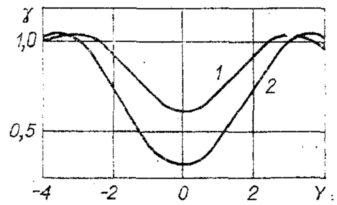

Fig. 2

$$\Psi'' = -\zeta \tag{1.4}$$

(the primes denote derivatives with respect to y). It can be shown that the vorticity perturbations in a thin CL are produced by the velocity field, which is clearly independent of them. The longitudinal velocity is given by the primary flow, the transverse is determined by vorticity perturbations outside the CL limits, and is found from the linear external equation (1.2): $\bar{u} - c \approx \bar{u}'_c\eta$, $v = -2\,\mathrm{Re}\,[i\alpha B \exp(i\alpha\xi)]$.* However, the jump in amplitude of the longitudinal velocity fluctuations, induced by vorticity perturbations in the CL, exerts substantial influence on the profile of the velocity fluctuations in the external flow domains (relative to the CL), which is also expressed by the second condition in (1.3). Precisely this jump determines the energy flux and momentum from the CL to the external domains. The first condition in (1.3) corresponds to a small increment in the transverse velocity during passage through the thin vortex layer.

Limiting ourselves to the case of a flow with monotonic velocity growth $[u'(y) > 0]$ and representing the CL vorticity in the form $\zeta = -\bar{u}'_c + (-\bar{u}''_c\eta + \zeta_i)$, we introduce the normalized variables

$$X = d\xi, \quad Y = (y - y_c)/d_l, \quad \Omega = \zeta_i/\bar{u}''_c d_l, \quad s = d_n^2/d_l^2,$$

where s is the normalized wave amplitude (for $s \sim 1$ the viscosity and nonlinearity effects in the CL are equalized). Since the case of large s is not examined later, the normalizations are made in terms of the thickness d_ℓ of a viscous CL. Without loss of generality, we can consider the amplitude B to be real and positive. Then an equation of the form

$$\partial^2\Omega/\partial Y^2 - Y\partial\Omega/\partial X - s\sin X\,\partial\Omega/\partial Y = -s\sin X \tag{1.5}$$

follows from (1.1a). It differs from the analogous equation obtained in [9] by the method of mergeable asymptotic expansions only in the mode of writing (see (3.20) in [9]). The desired solutions (1.5) should describe damping of the vorticity oscillations for $|Y| \gg 1$. Asymptotics of such solutions (1.5) can be represented as expansions in powers of 1/Y:

$$\Omega \to C \pm (1/2)H - (s/Y)\cos X + (s^2/4Y^3)\cos 2X + \ldots, \tag{1.6}$$

where C is an arbitrary constant, H is the jump in the mean vorticity induced by the nonlinear CL. The need to include H in (1.6) becomes evident after the average of (1.5) is taken over the wave period, and integration of the equation obtained in Y (see [9] for details). Using (1.2) and (1.4), the jump in phase of the logarithm Φ can be related to the solution of the boundary-value problem (1.5) and (1.6):

$$\Phi = -(2/s)\int_{-\infty}^{\infty} \langle\Omega \sin X\rangle\, dY, \tag{1.7}$$

where <...> denotes the mean wave over the period. The dependence of Φ on the parameter $\lambda_c = d^3_\ell/d^3_n \equiv s^{-2/3}$ is constructed in [9]. A simple analytic approximation $\Phi = -\pi/(1 + 0.68s^2)^{3/4}$ is proposed for it in [12]. The normalized curvature of the mean velocity profile in the CL, $\gamma = \langle u\rangle''/\bar{u}''_c = 1 - d\langle\Omega\rangle/dY$, is shown in Fig. 2 (curves 1 and 2 correspond to s = 1 and 2). It is seen that the profile of the mean velocity in the CL tends to the linear as the wave amplitude increases.†

*This can be seen by using the solution (1.2) for $|\eta| \geq d_c$ and estimating the velocity increment in the layer $|\eta| < d_c$ from (1.4) and the condition $\zeta(\eta) \sim \zeta|_{\eta=\pm d_c}$.

†The problem (1.5), (1.6) was solved numerically to determine the CL structure. The solu-

2. TAKING ACCOUNT OF THE INFLUENCE OF THE WALL

To use the bypassing rule (1.3) in the presence of a wall, it is required that the solution of (1.5) emerges on the asymptotic (1.6) prior to intersection with the wall level. This condition for the isolation of the CL from the wall can be written in general form as $y_c d_c \gg 1$. Moreover, because of fluid adherence to the wall, perturbations of nonviscous type (1.2) generate viscous vortical perturbations that damp out rapidly in directions from the wall, a viscous near-wall layer originates. It was emphasized in [7] that for moderately high Reynolds numbers, the process of transforming the viscous perturbations during passage through the CL exerts strong influence on the shape of the neutral curve. This process is substantial even for $y_c/d_\ell \equiv z_c \geqslant 3$, when the CL can be considered isolated. Neglecting the shift in the primary flow velocity in determining the VNWL structure, and considering the CL isolated, only the asymptote of the upper branch of the neutral curve can be obtained for very large RE [9, 12]. The approximation of an isolated CL with the transformation of viscous perturbations in the CL taken into account is actually utilized in linear instability theory and permits construction of the fundamental part of the boundary layer neutral curve loop [1, 7, 14, 15].

Let us show that an analogous approximation can be used even in the case of a nonlinear CL. Analysis of the nonlinear problem is complicated by the fact that the mean flow due to the jump in vorticity in the CL (see Sec. 1) is only found uniquely with the fact of the primary flow not being parallel taken into account. The asymptotic expansions should correspondingly contain the ratio $x/\mathrm{Re} \ll 1$ [9] as the small parameter. However, a simple scheme based on graphic physical representations and being a generalization of the Tollmien method known in linear theory [1] can be proposed to seek the eigenvalues of the boundary-value problem in the principal approximation.

In the case of an isolated CL the viscous perturbations damp out strongly on the path from the wall to the coincidence layer.* Consequently, they exert weak influence on the velocity field in the CL (see Sec. 1) and (1.5) remains valid. It is seen that (1.5) describes not only the process of generation of vortical perturbations on the vorticity gradient of the primary flow, but also the process of propagation of viscous type perturbations incident on the CL. To take account of the viscous perturbations arriving from the wall, we set $\Omega = \Omega^{(1)} + \Omega^{(2)}$, where $\Omega^{(1)}$ is the forced solution of (1.5) examined in Sec. 1, and $\Omega^{(2)}$ is the solution of the homogeneous equation

$$\frac{\partial^2 \Omega^{(2)}}{\partial Y^2} - Y\frac{\partial \Omega^{(2)}}{\partial X} - s\sin X\frac{\partial \Omega^{(2)}}{\partial Y} = 0 \tag{2.1}$$

that satisfies the damping condition for the vorticity perturbations above the CL: $\Omega^{(2)} \to$ const as $Y \to +\infty$. It can be shown that the nature of the asymptotic behavior of $\Omega^{(2)}$ as $|Y| \to \infty$ is determined by an operator consisting of the first and second terms in (2.1). Only the component $\Omega^{(1)}$ that governs merging of the inviscid solutions (1.2) will enter into the expression for the jump in the phase of the logarithm (1.7). Equation (1.2) can be used to describe viscous perturbations in the whole range between the wall and the CL. Indeed, for $s = 0$, an equation known in linear theory [1] follows for the complex amplitude of the harmonic [$\sim\exp(iX)$] perturbation. If $s \neq 0$, the contribution of the term s in (2.1) diminishes for $s/\sqrt{Y}$ as $|Y| \to \infty$. Therefore, even in the case of viscous perturbations the nonlinearity turns out to be localized in the CL. Consequently, the distinction between the fixed transverse velocity fields used in deducing (1.5) and (2.1) and the true distribution near the wall (which vanishes at the wall) results in a small error within the limits of applicability of this approach.

Let us introduce a stream function for the viscous perturbations $\tilde{\psi}$ and transform from $\Omega^{(2)}$ to vorticity in the initial normalization $\tilde{\zeta}$. Since there is also the relationship (1.4) between $\tilde{\psi}$ and $\tilde{\zeta}$, the condition that the longitudinal velocity vanish at the wall yields a boundary condition for (2.1)

---

tion was represented in the form of a segment of a complex Fourier series (see Sec. 2 analogously). Good agreement was obtained with the values of Φ presented in [9].

*The estimate $\ell_W \sim y_c\sqrt{2}(d_\ell/y_c)^{3/2} \ll y_c$ can be obtained for the scale of viscous perturbation diminution along y, the direction near the wall.

$$\int_{\infty}^{0} \tilde{\zeta}_\alpha dy = \psi'_\alpha|_{y=0}, \tag{2.2}$$

where α is the complex amplitude of the Fourier harmonic: $\psi_\alpha = \langle\psi \exp(-i\alpha\xi)\rangle$, etc. To take account of the reverse influence of the viscous perturbations on the inviscid perturbations (1.2), we write the condition that the normal velocity component is zero at the wall

$$\psi_\alpha = \int_{\infty}^{0} dy_1 \int_{\infty}^{y_1} dy_2 \tilde{\zeta}_\alpha(y_2)|_{y=0}. \tag{2.3}$$

From (2.2) and (2.3) we obtain the boundary condition for ψ_α in the form

$$\psi_\alpha = -y_c F \psi'_\alpha|_{y=0}, \tag{2.4}$$

where

$$F = -\int_{\infty}^{-y_c} d\eta_1 \int_{\infty}^{\eta_1} d\eta_2 \tilde{\zeta}_\alpha / y_c \int_{\infty}^{-y_c} d\eta_1 \tilde{\zeta}_\alpha$$

characterizes the effective pliability of the boundary for the inviscid perturbations.

To compute F we set the vorticity normalization $\tilde{\zeta}$ in correspondence with the boundary condition (2.2) by setting $\tilde{\Omega} = \tilde{\zeta}(2d\ell/u_m)$, where $u_m = 2|\psi'\alpha(0)|$ is the amplitude of the longitudinal velocity fluctuations on the wall in the inviscid solution (1.2). Representing $\tilde{\Omega}$ in the form of a Fourier series

$$\tilde{\Omega} = \sum_{n=-\infty}^{\infty} \tilde{\Omega}_n \exp(inX)$$

($\tilde{\Omega}_0 = \langle\tilde{\Omega}\rangle$, $\tilde{\Omega}_{-n} = \tilde{\Omega}^*_n$), we obtain a system of equations for the complex amplitudes of the harmonics in the form

$$\frac{\partial^2 \tilde{\Omega}_n}{\partial Y^2} - inY\tilde{\Omega}_n = \frac{1}{2} is \left(\frac{\partial \tilde{\Omega}_{n+1}}{\partial Y} - \frac{\partial \tilde{\Omega}_{n-1}}{\partial Y} \right). \tag{2.5}$$

The boundary conditions for $\tilde{\Omega}_n$ for n = 1, 2, 3, ... have the form

$$\int_{\infty}^{-z_c} \tilde{\Omega}_n dY = E\delta_{1n}, \quad \tilde{\Omega}_n \to 0|_{Y\to\infty}, \tag{2.6}$$

where δ_{ij} is the Kronecker delta; $E = \exp(i\theta)$; $\theta = \arg[\psi'_\alpha(0)/\psi_\alpha(y_c)]$ is the shift between the phases of the stream function fluctuations in the CL and the longitudinal velocity at the wall in the inviscid solution (1.2). Taking into account that $Y \to \infty$, $d\tilde{\Omega}_0/dY \to 0$, it is possible to eliminate $d\tilde{\Omega}_0/dY$ from the system (2.5) and obtain a closed boundary-value problem for $\tilde{\Omega}_n$ with n = 1, 2, ... We represent the solution of this problem in the form $\tilde{\Omega}_n = \Omega^{(c)}{}_n \cos\theta + \Omega^{(s)}{}_n \sin\theta$, where $\Omega^{(c)}{}_n$ satisfied (2.5) and (2.6) for E = 1 and $\Omega^{(s)}{}_n$ is the solution of (2.5) and (2.6) for E = i. Correspondingly, we obtain an expression for F:

$$F(z_c, s; \theta) = (F_c \cos\theta + F_s \sin\theta)e^{-i\theta},$$

$$F_a = -\frac{1}{z_c} \int_{\infty}^{-z_c} dY_1 \int_{\infty}^{Y_1} dY_2 \Omega_1^{(a)}(Y_2) \quad (a = c, s).$$

In the limiting case of linear waves ($s \to 0$), the relationship $\Omega^{(s)}{}_1 = i\Omega^{(c)}{}_1$ is satisfied. Here F is independent of s, θ and agrees with the function $F(z_c)$, known well from linear stability theory [14, 15]. For $s \neq 0$ the boundary-value problem (2.5) and (2.6) was solved numerically. The computations were made for $s \leqslant 2$ with the first four harmonics retained in the Fourier expansion. The results of calculating F_c and F_s for $z_c = 3.8$ are represented in Table 1.

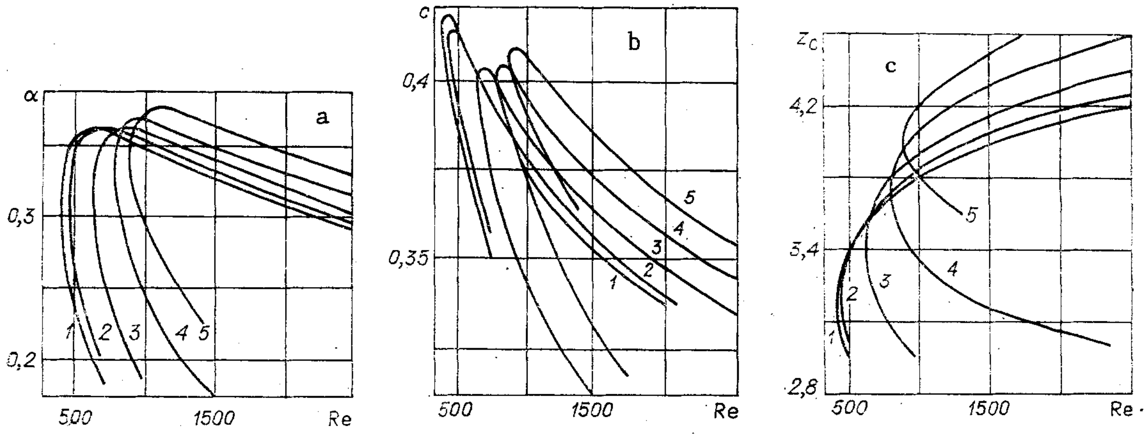

Fig. 3

TABLE 1

s	Re F_c	Im F_c	Re F_s	Im F_s
0	0,2036	0,3213	—0,3213	0,2036
0,4	0,2081	0,3012	—0,3216	0,2067
1,0	0,2430	0,2258	—0,3218	0,2171
1,5	0,3018	0,1564	—0,3176	0,2217
2	0,3738	0,0934	—0,3039	0,2172

3. STATIONARY FLUCTUATIONS AND BUILDUP OF SELF-OSCILLATIONS

To close the boundary-value problem we use the standard condition on the boundary-layer $y = y_m$ [1]:

$$\psi_\alpha' + \alpha\psi_\alpha = 0\,|_{y=y_m}. \tag{3.1}$$

Substituting (1.2) into (2.4) and (3.1) results in the following characteristic equation for the stationary pulsations:

$$F(z_c, s;\, \theta) = \frac{1}{y_c}\,\frac{\varphi_a(0)\,\Delta_b - \varphi_b(0)\,\Delta_a - i\Phi\frac{\bar{u}_c''}{\bar{u}_c'}\,\varphi_a(0)\,\Delta_a}{\varphi_b'(0)\,\Delta_a - \varphi_a'(0)\,\Delta_b + i\Phi\frac{\bar{u}_c''}{\bar{u}_c'}\,\varphi_a'(0)\,\Delta_a}, \tag{3.2}$$

where $\Delta_{a,b} = \varphi_{a,b}' + \alpha\varphi_{a,b}|_{y=y_m}$. It can be shown that (3.2) goes over into the characteristic equation of linear theory [1] as $s \to 0$. Let us also note here that in the case when the resonance interaction of the wave-flow and the formation of the VNWL, being taken into account separately yield weak damping (amplification) of the wave, the amplitude of the stationary fluctuations can be found from the condition that the gain increment equals the damping decrement [12, 16]. However, this graphic representation is only applicable for sufficiently small α, which is expressed in the significant shift of the point for the loss of stability in the limit case of linear waves [16].

To evaluate the roots of (3.2) for $s \neq 0$ it is necessary to determine θ for "trial" solutions (1.2). It is convenient to take θ from the solution (1.2) satisfying the boundary condition (3.1) and the rule for bypassing the resonance point (1.3). The system of two real equations that follows from the complex equation (3.2) was solved numerically for α and c for fixed z_c and s. The functions φ_a and φ_b were sought by numerical integration of the Rayleigh equation (see [8] analogously). The computations were made for a boundary layer with a Blasius velocity profile. The thickness δ (see Sec. 1) was taken equal to the boundary-layer displacement thickness, and correspondingly we set $y_m = 3$. The Blasius profile $\bar{u}(y)$

was approximated by a piecewise-linear function given by 3000 identical segments in the interval $0 \leq y \leq 3$. The results of the calculations are represented in Fig. 3a-c in the form of level curves for the stationary fluctuation amplitudes on different planes of the parameters (curves 1-5 correspond to s = 0, 0.4, 1, 1.5, and 2).

The neutral curves of linear theory to which the lines s = 0 correspond in Fig. 3 have the loss-of-stability point $Re_n \approx 420$, which is in agreement with the results of computations for a rougher velocity profile approximation [17]. The intersection of the line s = const and the neutral curve is shifted to the tip of the neutral curve as s diminishes. A weakly nonlinear theory, in which the limit position of this point (Re_*) corresponds to disappearance of the second coefficient in the Landau equation [6], results in the same deduction. The behavior of the amplitude level curve on the plane (Re, α) can be related to the wave development in time when the wave number α is conserved. It is natural to assume that hard excitation of self-oscillations is realized in that domain of the parameters Re, α, where the level lines intersect (it is bounded by the upper branch of the linear theory neutral curve and the envelope of the family s = const). Here the greater of the two values of s corresponds to the self-oscillation regime, while the smaller characterizes the threshold of the hard excitation of self-oscillations. Soft excitation holds within the neutral curve loop, where there is no intersection of the level lines. These assumptions are confirmed by the results of solving the nonstationary problem in different limit cases [6, 12, 16]. Let us emphasize that self-oscillatory regimes with a small level of nonlinearity ($s \ll 1$) are possible only near the lower part of the neutral curve loop (soft excitation) adjoining the limit point Re_* and in a small neighborhood of this point (hard excitation).

The approximation of an isolated CL loses meaning for large-amplitude waves when the closed streamlines of the velocity fields in the CL ("cat's eye" [8]) approach the wall. Since removal of the cat's eye boundary from the coincidence layer $y = y_c$ equals $2d_n$, we obtain a constraint on the wave amplitude in the form $s < {}^1/_4 z^2{}_c$. This condition is satisfied for the curves shown in Fig. 3. As seen from Fig. 3, the amplitude of the self-oscillatory regimes grows rapidly during motion along the upper branch of the neutral curve s = 0 toward large Re, which is explained by attenuation of the influence of the CL nonlinearity on the efficiency of the viscous wave destabilization mechanism. For very large Reynolds numbers, when the viscous mechanism becomes linear because of the rapid damping of the viscous perturbations on the path from the wall to the CL, the approximation of an isolated CL yields only those waves which correspond to the threshold of hard origination of instability [9, 12]. Therefore, in the case of an isolated CL stabilization of the two-dimensional instability is associated with the influence of the CL nonlinearity on both the efficiency of resonance wave-flow interaction and the efficiency of the viscous wave destabilization mechanism.

Buildup of the self-oscilalting regime in the two-dimensional stage of instability development in the boundary layer on a plate was observed in [18]. The amplitude of the longitudinal velocity fluctuations in their maximum on the wave profile originating near the wall was here approximately 2.8% of u_∞. This quantity is close to the amplitude of the longitudinal velocity fluctuation on a wall in the inviscid solution (1.2), u_m (see Sec. 2) in the theory developed above. For a wave with amplitude s = 1 with Re ~ 800 that corresponds, as in [18], to the self-oscillatory regime in the neighborhood of the neutral curve upper branch, calculations yield $u_m/B \approx 1.66$. Hence the value $u_m \approx 3.1\%$ is obtained, which is close to that measured in [18].

In conclusion, let us note that the accuracy of the asymptotic theory, within which the characteristic equation is constructed for the nonlinear waves (3.1), permits only a rough comparison between the computation results and the experimental data. As is known, even in a linear approximation the asymptotic theory results in a noticeable quantitative divergence from the results of direct numerical integration of the Orr–Sommerfield problem. Moreover, for moderately large Re the corrections associated with the flow not being parallel [3] play a noticeable role. At the same time, the asymptotic approach permits giving a qualitative analysis of the wave field structure as a whole. In this paper, the mechanism of the limitation of the two-dimensional instability is clarified with its aid, and the nature of the dependence of the self-oscillation amplitude and the threshold of their hard origination on the wave parameters and Reynolds number is determined. The analysis performed is also of interest for the construction of models of three-dimensional nonlinear structures occurring in the transition region.

LITERATURE CITED

1. H. Schlichting, Boundary Layer Theory [Russian translation], Nauka, Moscow (1974).
2. R. Betchov, "Transition." Turbulence. Principles and Applications [Russian translation], Mir, Moscow (1980).
3. Yu. S. Kachanov, V. V. Kozlov, and V. Ya. Levchenko, Origin of Turbulence in the Boundary Layer [in Russian], Nauka, Novosibirsk (1982).
4. S. A. Maslow, "Instability in shear flows and transition," Hydrodynamic Instability and Transition to Turbulence [Russian translation], Mir, Moscow (1984).
5. W. Reynolds and M. S. Potter, "Finite-amplitude instability of parallel shear flows," Mekhanika, No. 2 (1968).
6. S. Ya. Gertsenshtein and Yu. M. Shtemler, "Nonlinear perturbation development in boundary layers and their stability," Dokl. Akad. Nauk SSSR, 234, No. 6 (1977).
7. R. Betchov and W. Criminale, Jr., Questions of Hydrodynamic Instability [Russian translation], Nauka, Moscow (1971).
8. D. J. Benney and R. F. Bergeron, "A new class of nonlinear waves in parallel flows," Stud. Appl. Math., 48, No. 3 (1969).
9. R. Huberman, "Critical layers in parallel flows," Stud. Appl. Math., 51, No. 2 (1972).
10. V. N. Zhigulev, "State of the art of laminar flow instability problems," in: Mekhanika Turbulentnykh Potokov [in Russian], Nauka, Moscow (1980).
11. A. V. Zubtsov and V. I. Ponomarev, "On a class of nonstationary waves in an incompressible fluid boundary layer," in: Aeromekhanika [in Russian], Nauka, Moscow (1976).
12. V. P. Reutov, "Nonstationary critical layer and nonlinear stage of instability in plane Poiseuille flow," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1982).
13. K. M. Case, "Hydrodynamic instability as a problem with initial data," in: Hydrodynamic Instability [Russian translation], Mir, Moscow (1964).
14. C.C. Lin, Theory of Hydrodynamic Instability [Russian translation], IL, Moscow (1958).
15. A. M. Basin, A. I. Korotkin, and L. F. Kozlov, Ship-Boundary Layer Control [in Russian], Sudostroenie, Leningrad (1968).
16. V. P. Reutov, "Nonlinear stage of two-dimensional instability in the boundary layer," Preprint Inst. Prikl. Fiz. Akad. Nauk SSSR, No. 57, Gor'kii (1982).
17. N. E. Kochin, M. A. Kibel', and N. V. Roze, Theoretical Hydromechanics [in Russian], Part 2, Gostekhizdat, Moscow (1948).
18. Yu. S. Kachanov, V. V. Kozlov, and V. Ya. Levchenko, "Nonlinear wave development in a boundary layer," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3 (1977).